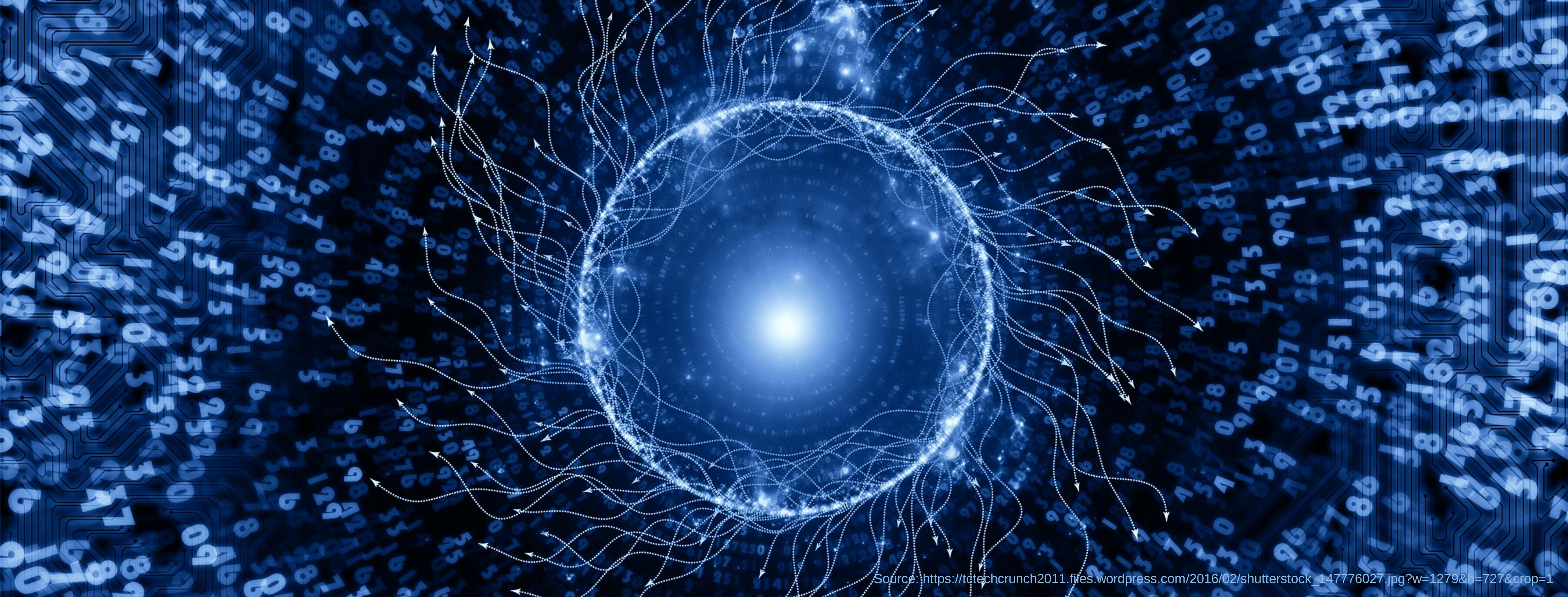


Source: https://tctechcrunch2011.files.wordpress.com/2016/02/shutterstock_147776027.jpg?w=1279&h=727&crop=1

# Robotics for Future Industrial Applications

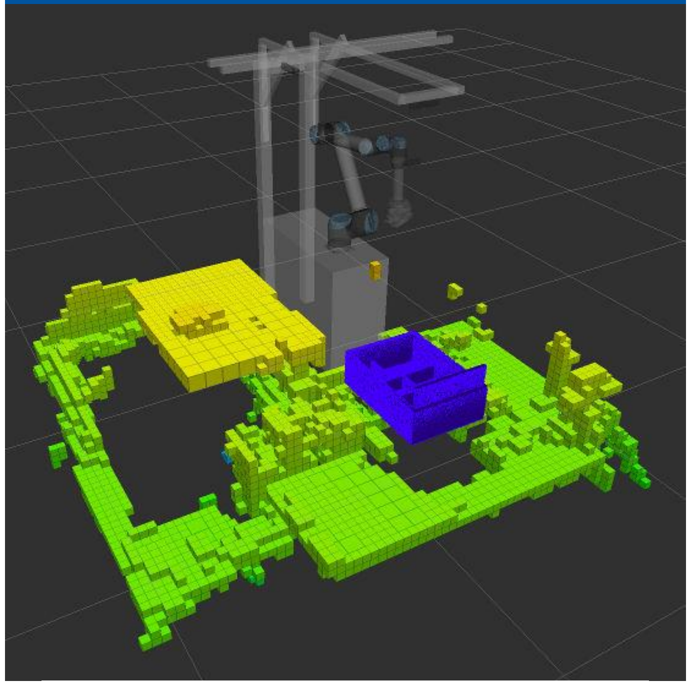
**Modelbased Reinforcement Learning**

Philipp Ennen, M.Sc.

IfU | IMA | ZLW | RWTH AACHEN UNIVERSITY

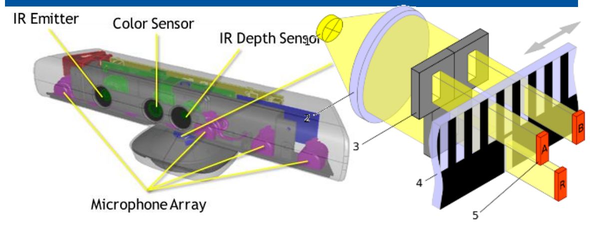
### How do I (the robot) go there?

**Model of the Robot and Environment**



$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}u$$

**Sensing**




→ **Motion Planning with iLQR** →

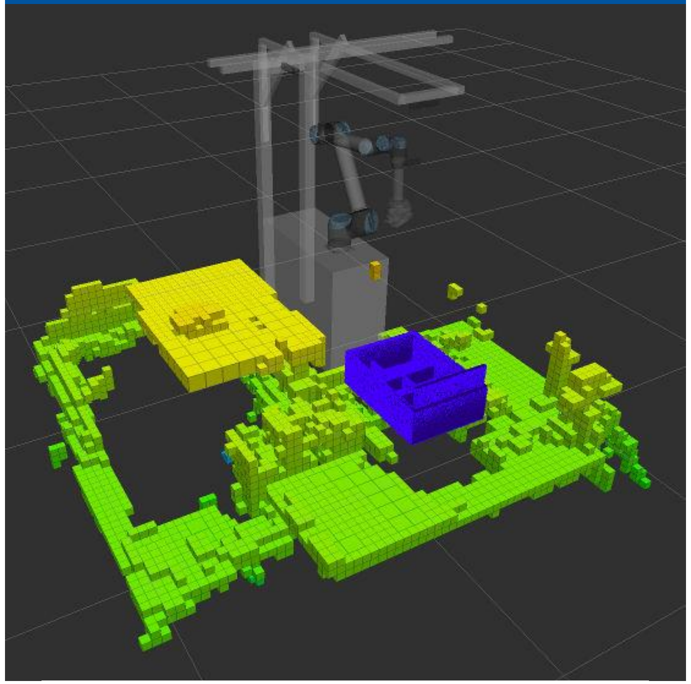
Requires Goalstate:
- i.e. hand-engineered
- i.e. via a cost function

**Motion Control and Execution**

# Introduction

## How do I (the robot) go there?

**Model of the Robot and Environment**



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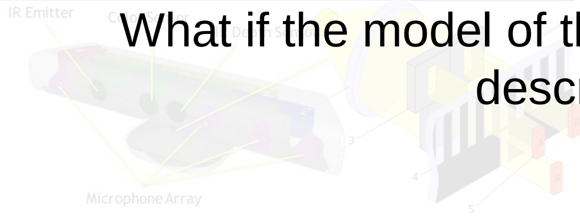
Sensing

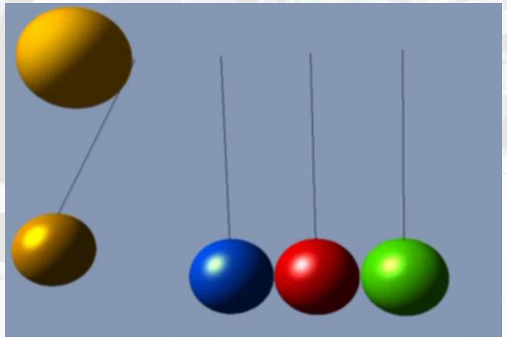



Motion Control and Execution

What if the model of the robot and environment is hard to describe (or unknown)?

Think about flexible objects!

Think about contact-situations!



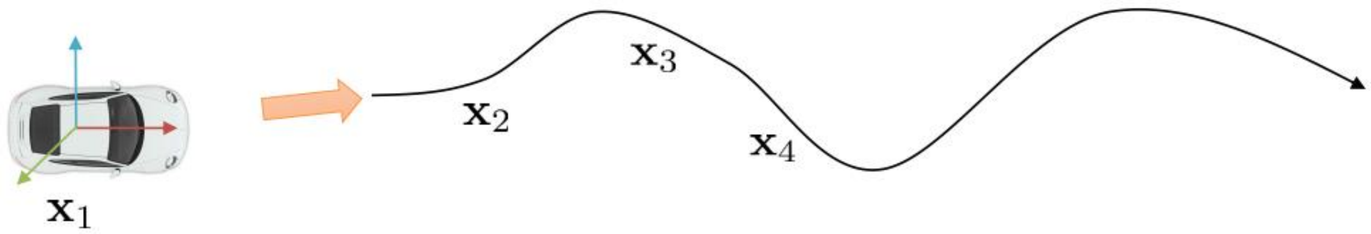
Requires Goalstate:
- i.e. hand-engineered
- i.e. via a cost function

## Dynamics of unstructured environments

## Real-World Dynamics are Complex!

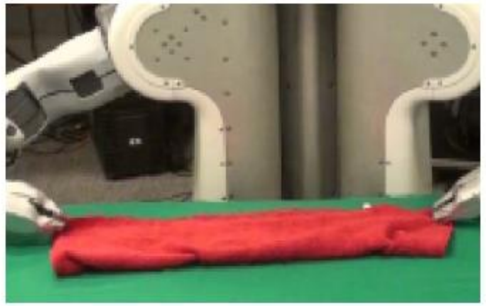
Dynamics of
a car

Often dynamic models exist




Dynamic models usually do not exist


Dynamics of
contacts



Dynamics of
flexible objects

Dynamics of
unstructured environments

## Trajectory Optimization

- Trajectory Optimization: Calculates optimal sequence of actions using cost-function and dynamics

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

- Today: How can optimal action sequences be calculated if a dynamic model does not exist?

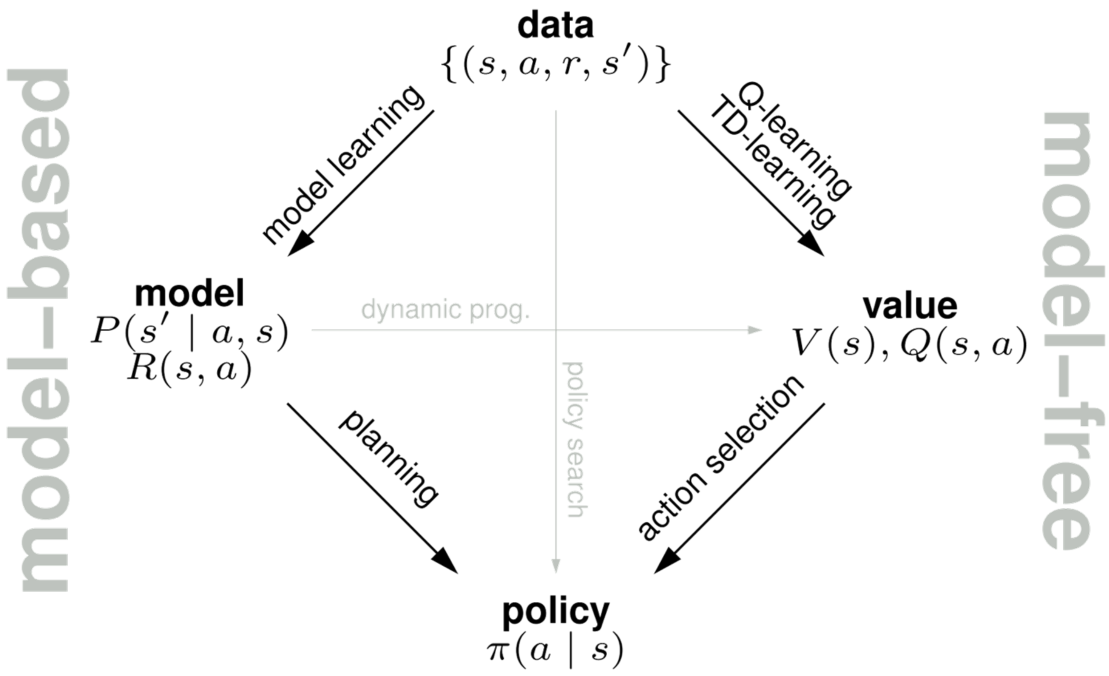
- Today we learn an algorithm based on
  – Learning a global dynamic model (“model-based reinforcement learning”)
  – Learning a local dynamic model

## What if the model of the robot and environment is hard to describe (or unknown)?

This weeks topic!

**Model-based RL:**

- Learn to predict next state (using a dynamic model): $P(s'|s,a)$
- ~~Learn to predict immediate reward $P(r'|s,a)$~~ (we assume to have this information)




**Model-free RL:**

- Learn to predict value: $V(s)$ or $Q(s,a)$

$s$: state
$a$: action
$r$: reward

# Content

I. **Modelbased Reinforcement Learning**
   I. Learning of dynamic models
   II. Learning of dynamic models and policies

II. **Representing a dynamic model**

III. **Global and local dynamic model**

IV. **Learning with local dynamic models with „Trust Regions“**

# Motivation

## Why do we want to learn the dynamics?

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1,}\mathbf{u}_{t-1})$$

⬍

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

Usual procedure: Differentiate via Backpropagation and optimize (i.e. iLQR)

Requires: $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}, \frac{\partial c}{\partial \mathbf{x}_t}, \frac{\partial c}{\partial \mathbf{u}_t}$

## Why do we want to learn the dynamics?

- If $\mathbf{x}_{t+1} = f(\mathbf{x}_{t,}\mathbf{u}_t)$ is known, we can do trajectory optimization
  - In the stochastic case $\mathrm{p}(\mathbf{x}_{t+1}|\mathbf{x}_{t,}\mathbf{u}_t)$

Learn $f(\mathbf{x}_{t,}\mathbf{u}_t)$ with subsequent backpropagation (i.e. iLQR)

**Modelbased Reinforcement Learning Version 0.5**

1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}_,\mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_{i,}\mathbf{u}_i) - \mathbf{x}_i' \right\|^2$
3. Backpropagate $f(\mathbf{x}_,\mathbf{u})$ and calculate sequence of actions (i.e. iLQR)

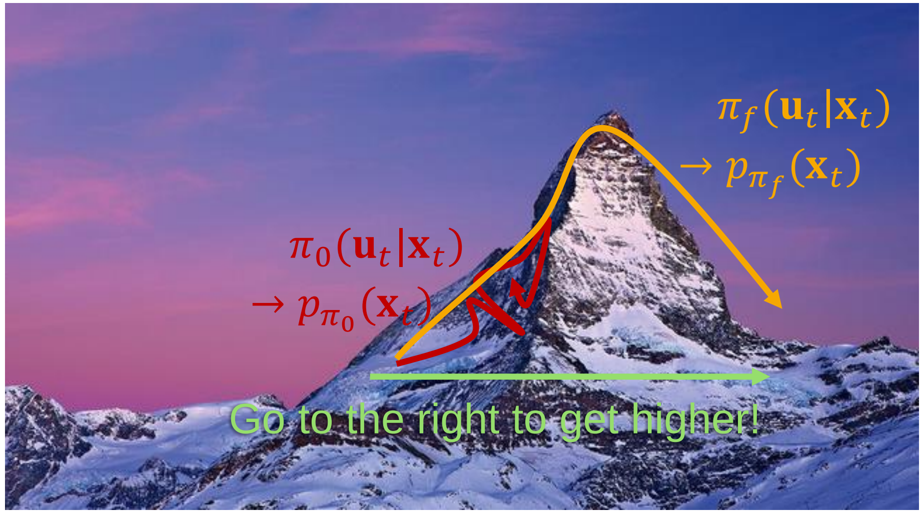
## Does Version 0.5 work?

(often) **YES!**

- Traditional system identification uses this method (control theory)
- Initial policy must be chosen with caution
- Version 0.5 is very effective
  - If a representation of the dynamics based on physical laws exists
  - If only a few parameters must be learned

## Does Version 0.5 work?

(in general) **NO!**




1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \| f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i \|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR) → $\pi_f(\mathrm{u}_t|\mathrm{x}_t)$

$$p_{\pi_0}(\mathbf{x}_t) \neq p_{\pi_f}(\mathbf{x}_t)$$

(Distribution Mismatch Problem)

❯ Distribution Mistmatch Problem increases if expressive classes of models are used (i.e. neural networks)

### Can we do better?

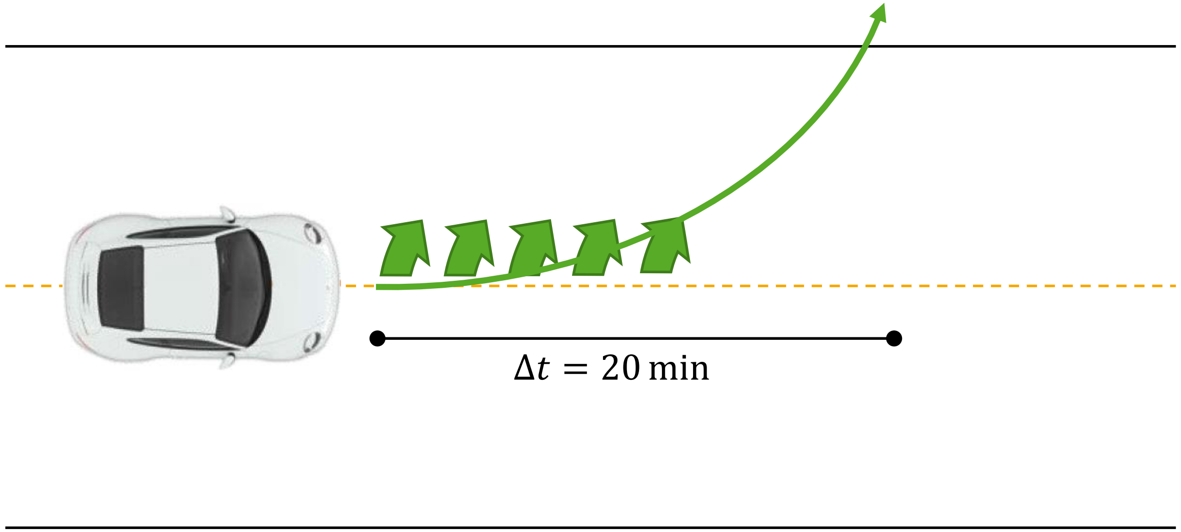
Can we make $p_{\pi_0}(\mathbf{x}_t) = p_{\pi_f}(\mathbf{x}_t)$?

Need to collect data from $p_{\pi_f}(\mathbf{x}_t)$!

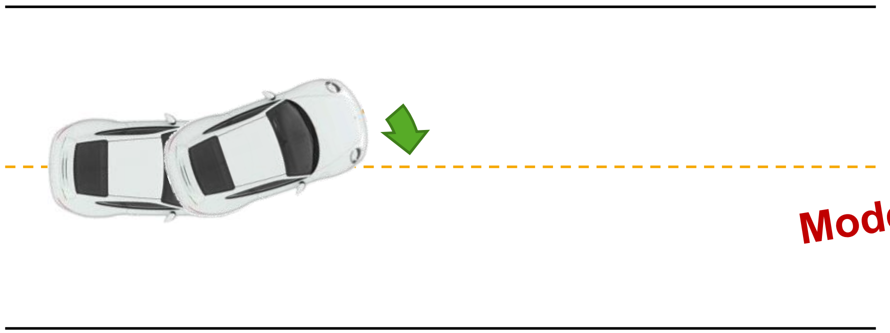
**Modellbasiertes Reinforcement Learning Version 1.0**

1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizesLearn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute those actions and add the resulting data $\{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$ to $\mathcal{D}$

## What happens if the dynamic models contains little error?



## Can we do better?



**Modelpredictive control helps with error**

### Modellbasiertes Reinforcement Learning Version 1.5

1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizesLearn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \|f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i\|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute the first planned action, observe resulting state $\mathbf{x}'$(MPC)
5. Append $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$ to dataset $\mathcal{D}$

every N steps

## Summary

- Version 0.5: collect random samples, train dynamics, plan
  - Pro: simple, no iterative procedure
  - Con: distribution mismatch problem

- Version 1.0: iteratively collect data, replan, collect data
  - Pro: simple, solves distribution mismatch
  - Con: open loop plan might perform poorly, exp. in stochastic domains

- Version 1.5: iteratively collect data using MPC (replan in each step)
  - Pro: robust to small model errors
  - Con: comoputationally expensive, but have planning algorithm available

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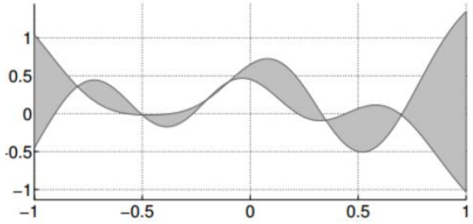
# What kind of models can we use?

### Gaussian process



GP with input $(\mathbf{x}, \mathbf{u})$ and output $\mathbf{x}$'

Pro: very data-efficient

Con: not great with non-smooth dynamics

Con: very slow when dataset is big

### Neural Network

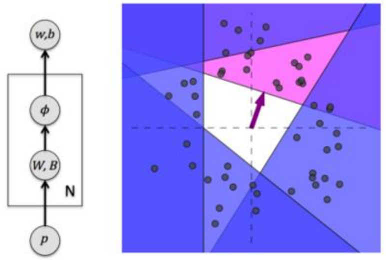


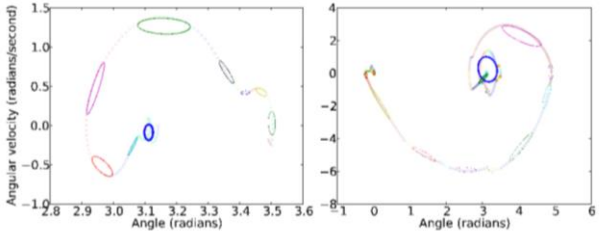

image: Punjani & Abbeel '14

Input is $(\mathbf{x}, \mathbf{u})$, output ist $\mathbf{x}$'

Pro: very expressive, can use lots of data

Con: not so great I low data regimes

### Gaussian Mixture Model




GMM over $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$ tuples

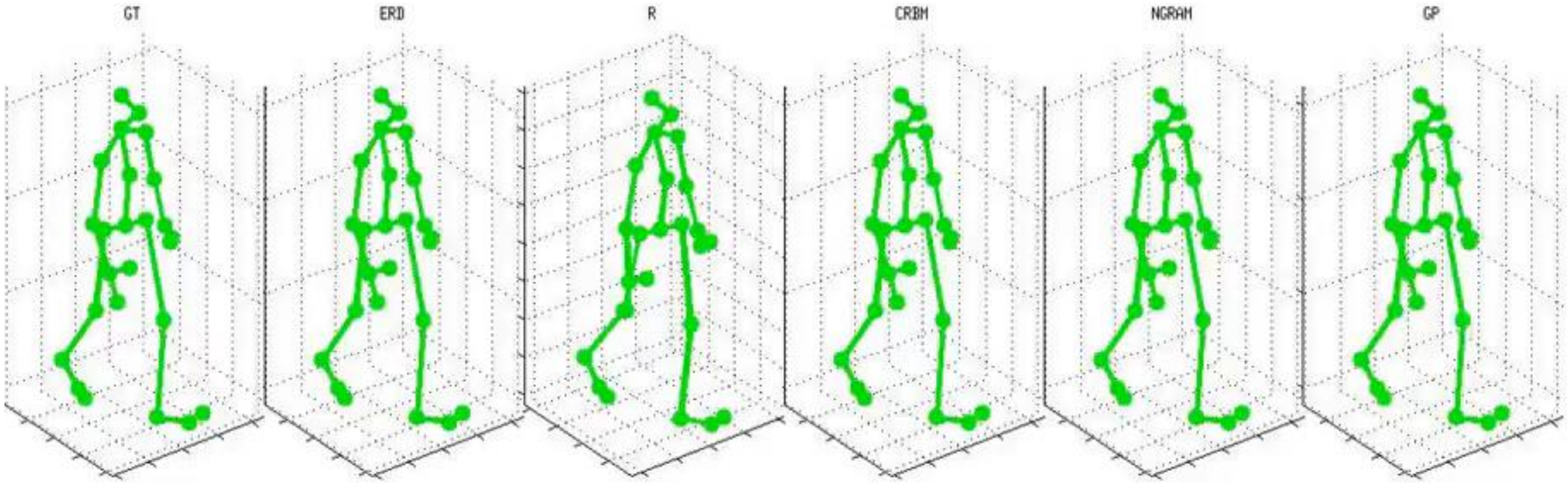
Train on $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$, condition to get $p(\mathbf{x}'|\mathbf{x}, \mathbf{u})$

For i'th mixture element, $p_i(\mathbf{x}, \mathbf{u})$ gives region where the mode $p_i(\mathbf{x}'|\mathbf{x}, \mathbf{u})$ holds

Pro: very expressive, if the dynamics can be assumed as piecewise linear

be assumed as piecewise linear

# Representation of Dynamic Models



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# Global Dynamic Models

## Challenges

Example: Global dynamic model $f(\mathbf{x}_t, \mathbf{u}_t)$ is represented by a neural network

**Modellbasiertes Reinforcement Learning Version 1.0**

1. Execute initial policy $\pi$_0 ($\mathbf{u}$_$t$ |$\mathbf{x}$_$t$) (i.e. a random policy) and collect data $\mathcal{D}$={($\mathbf{x}$,$\mathbf{u}$,$\mathbf{x}$^′ )_$i$}
2. Learn dynamics $f(\mathbf{x}_{,}\mathbf{u})$ that minimizesLearn dynamics $f(\mathbf{x}_{,}\mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_{i,}\mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}_{,}\mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute those actions and add the resulting data $\{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$ to $\mathcal{D}$

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution

# Global Dynamic Models

## The trouble with global models

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution
- In some tasks, the model is much more complex than the policy

## Motivation

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1,}\mathbf{u}_{t-1})$$

$$\Updownarrow$$

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

Usual story: differentiate via backpropagation and optimize (i.e. iLQR)

need: $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}$ $\frac{\partial c}{\partial \mathbf{x}_t}, \frac{\partial c}{\partial \mathbf{u}_t}$

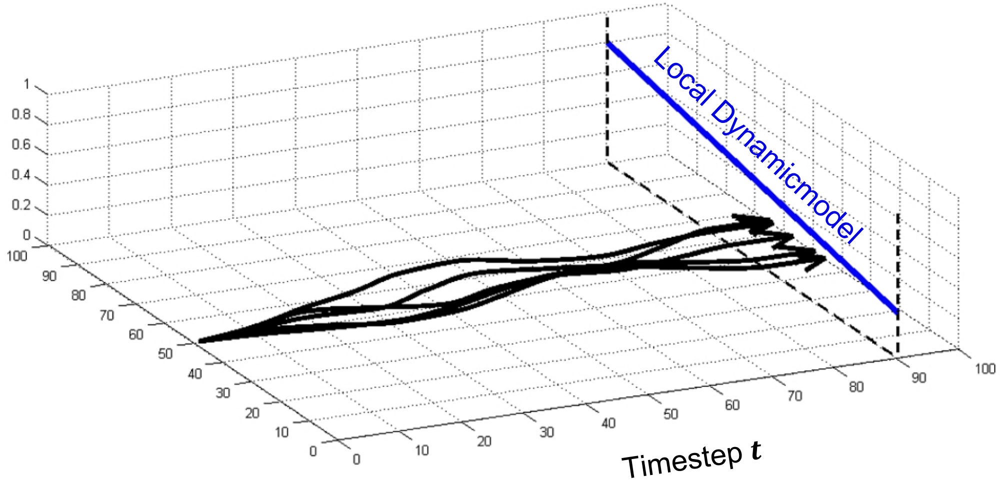
# Local models

## Approach

need: $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}, \frac{\partial c}{\partial \mathbf{x}_t}, \frac{\partial c}{\partial \mathbf{u}_t}$

idea: just fit $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}$ around current trajectory or policy

$p(\mathbf{u}_t|\mathbf{x}_t)$ – time-varying linear-Gaussian controller – can **execute** on the robot and produces **trajectory distribution**

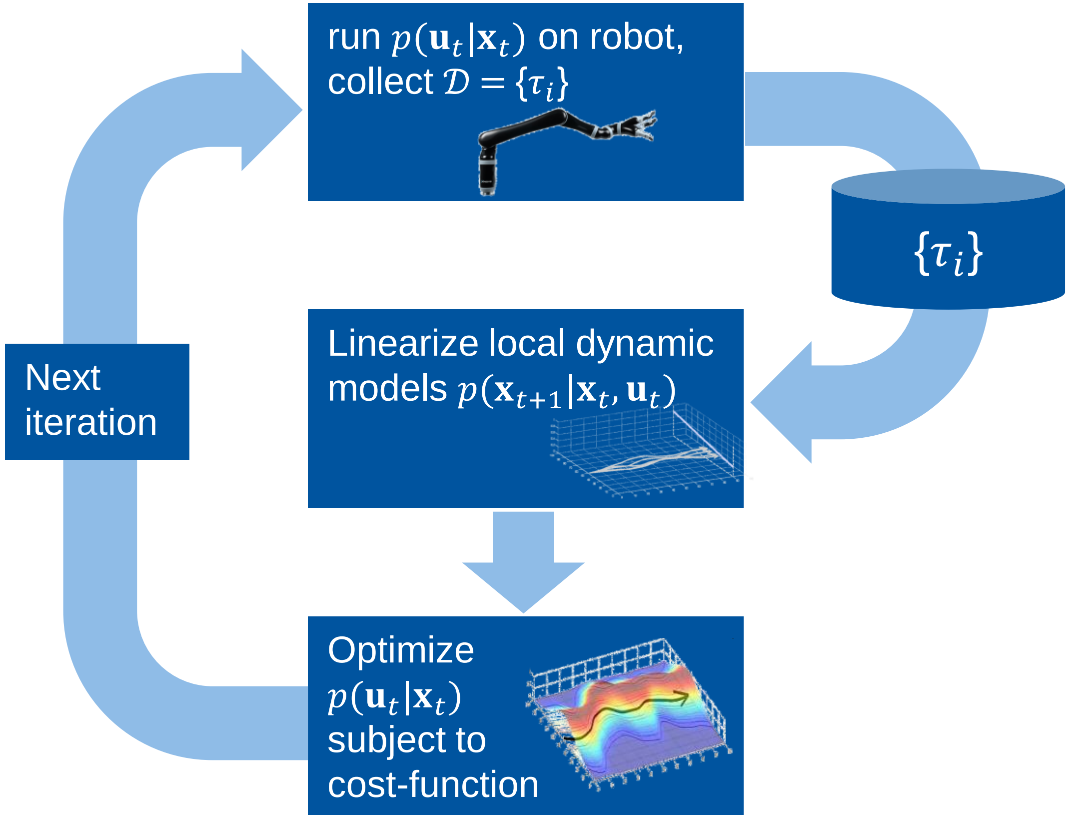


## Learning a policy

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

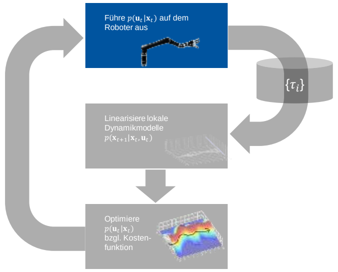
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{\partial f}{\partial \mathbf{x}_t} \qquad \mathbf{B}_t = \frac{\partial f}{\partial \mathbf{u}_t}$$



# Local models

## Timedependent Linear Gaussian Controller




run $p(\mathbf{u}_t|\mathbf{x}_t)$ on robot, collect $\mathcal{D} = \{\tau_i\}$

iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \mathbf{\Sigma}_t)$$

Set $\mathbf{\Sigma}_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

$Q(\mathbf{x}_t, \mathbf{u}_t)$ is the cost to go: total cost we get after taking an action $\mathbf{u}_t$

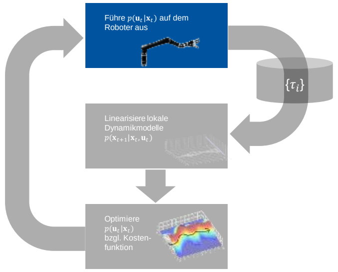
$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix} + \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{q}_t$$

$\mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}$ is big, if changing $\mathbf{u}_t$ changes the Q-value a lot!

If $\mathbf{u}_t$ changes Q-value a lot, don't vary $\mathbf{u}_t$ so much. Exploration noise $\mathbf{\Sigma}_t$ must be low

# Local models

## Timedependent Linear Gaussian Controller




run $p(\mathbf{u}_t|\mathbf{x}_t)$ on robot, collect $\mathcal{D} = \{\tau_i\}$

iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \boldsymbol{\Sigma}_t)$$

Set $\boldsymbol{\Sigma}_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

Standard LQR solves $$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t)$$

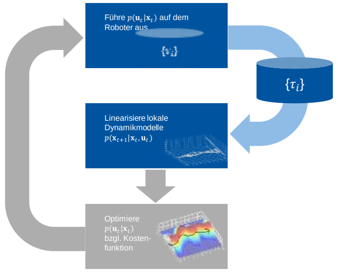
Linear-Gaussian solution solves $$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} E[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$$

Maximum Entropy: act as randomly as possible while minimizing cost

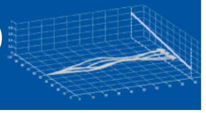
- Entropy: A measure for the average information content

# Local models

## Linearize local dynamics




Linearize local dynamic models
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$



$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

Version 1.0: Linearize $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}_t, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{\partial f}{\partial \mathbf{x}_t} \qquad \mathbf{B}_t \approx \frac{\partial f}{\partial \mathbf{u}_t}$$

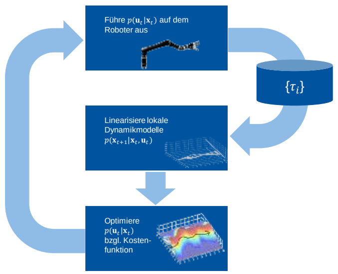
**Can we do better?**

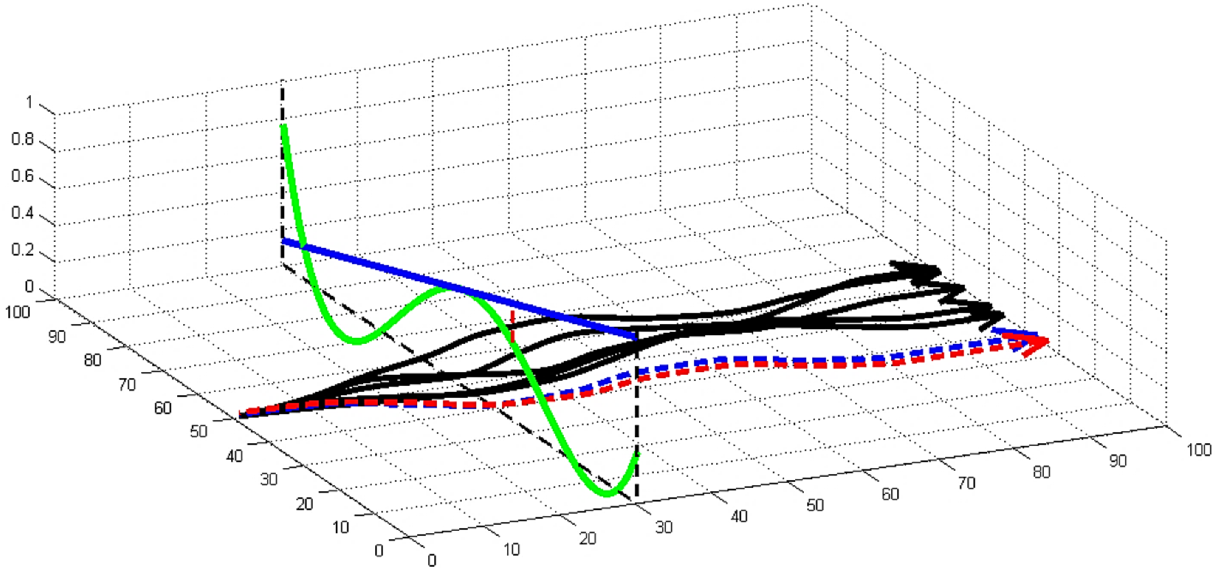
Version 2.0: Linearize $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ using *Bayesian* linear regression

– Bayesian linear regression uses prior: $p(\mathbf{x}_t, \mathbf{u}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = p(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})$
– Use your favourite global model as a prior (GP, deep net, GMM)

# Local models

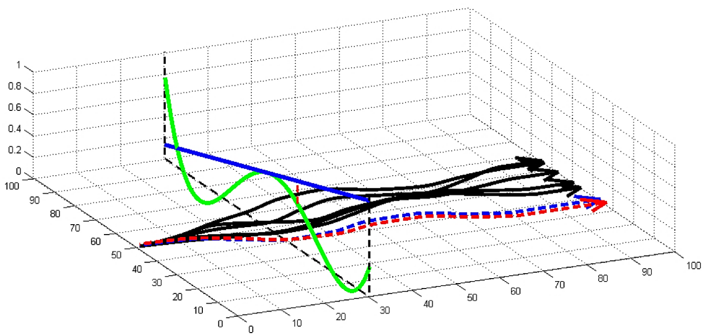
## How to stay close to old controller?



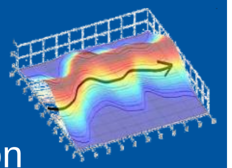



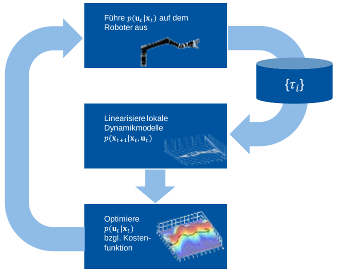
# Local models

## How to stay close to old controller?




Optimize
$p(\mathbf{u}_t|\mathbf{x}_t)$
subject to
cost-function






$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \mathbf{\Sigma}_t)$$

$$p(\tau) = p(\mathbf{x_1}) \prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t) p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

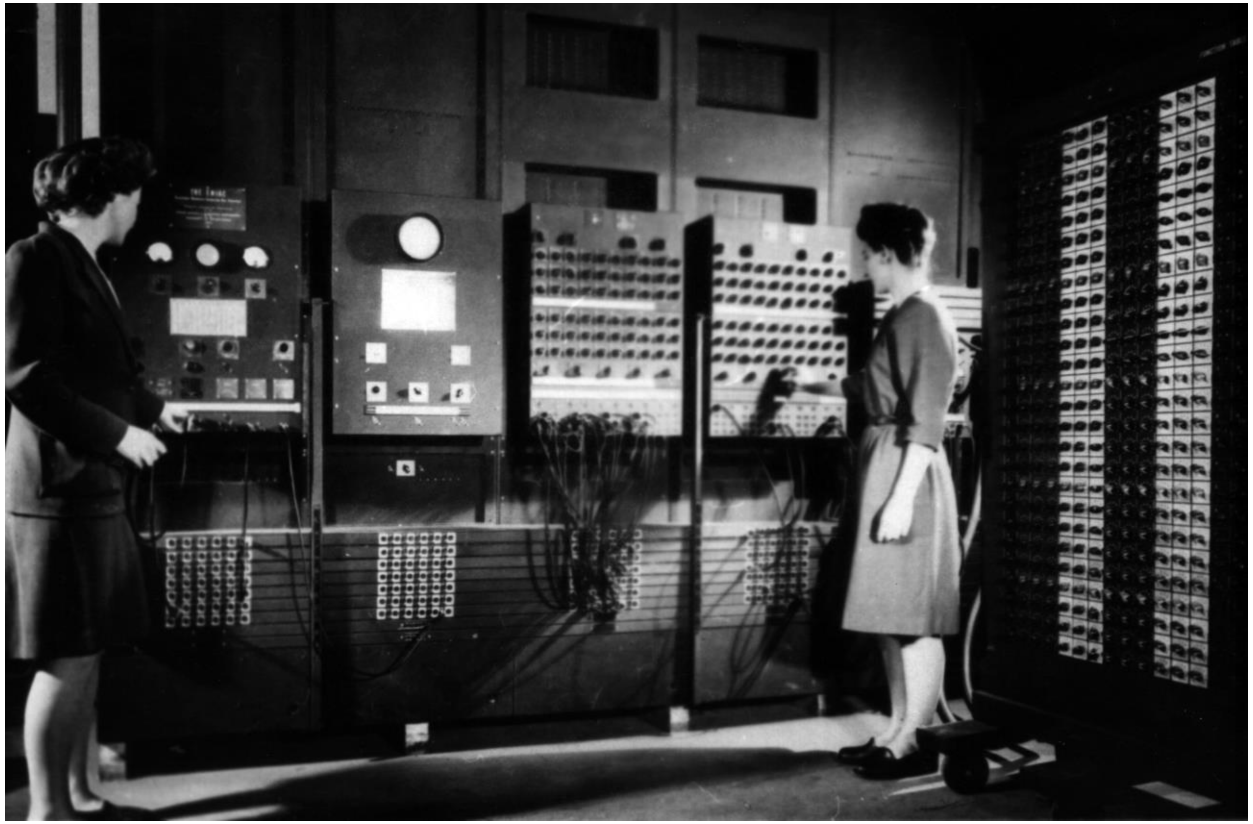
New trajectory distribution $p(\tau)$ must be similar to the old one $\bar{p}(\tau)$

If trajectory distribution is close, the dynamics will be close too!

What does “close” mean?
Kullback-Leibler divergence: $D_{KL}(p(\tau)||\bar{p}(\tau)) < \varepsilon$ **From here comes a lot of mathematics!**

# Its your turn!

## Visit the website and implement it!



# Introduction to the tasks

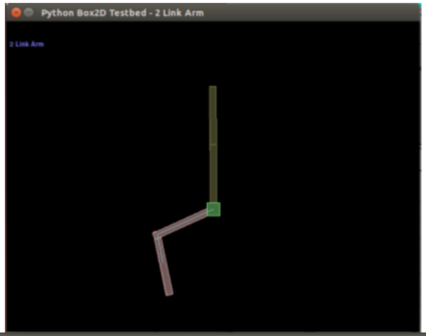
## Tasks for today and tomorrow

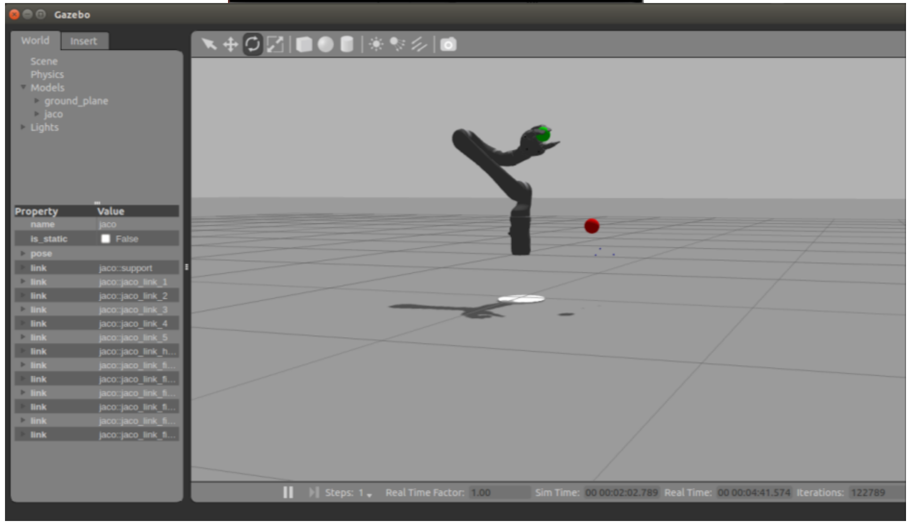
- Task 1:
  - Implement an LQR Backward and Forward pass
  - Try to understand it!
  - Test it with our test method

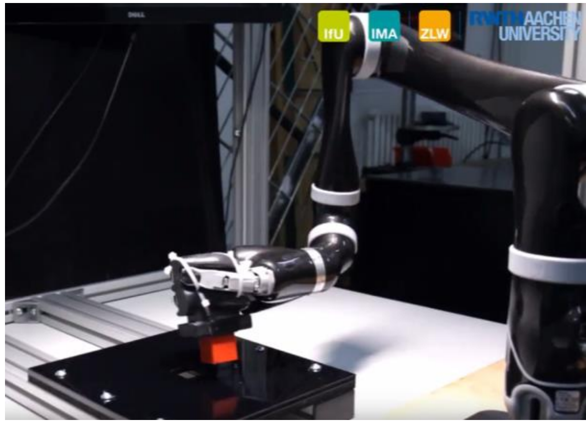
- Task 2:
  - Implement linearization of the dynamic model
  - Try to understand it!
  - Test it with our test-method
  - Test it on the Box2D Scenario

- Task 3:
  - Test it with Kinova Jaco 2 in simulation
  - Adjust cost function

- Task 4:
  - Test it with real Kinova Jaco 2
  - Adjust cost function



# Task 1 – Installation procedure

Download source code (do it in your home directory: `cd ~`):

```
git clone https://github.com/philippente/ss2017_task1_lqr.git
```

Edit .bashrc to set environment variables:

```
gedit ~/.bashrc
```

At the end of file, the lines should look like this:

```
source /opt/ros/indigo/setup.bash
source /home/useradmin/catkin_ws/devel/setup.bash
export
ROS_PACKAGE_PATH=$ROS_PACKAGE_PATH:/opt/ros/indigo/share:/opt/ros/indigo/stacks:/home/useradmin/ss2017_task1_lqr:/home/usera
dmin/ss2017_task1_lqr/src/gps_agent_pkg
```

Check if the blue part of the source folder and ROS_PACKAGE_PATH is correct!

Then save it and close it. Source the .bashrc (load the environment variables):

```
source ~/.bashrc
```

Now, compile some stuff:

```
cd ss2017_task1_lqr
sh compile_proto.sh
cd /src/gps_agent_pkg
cmake .
make -j
```

# Task 1 – Installation procedure

- Open PyCharm
- Import the folder *ss2017_task1_lqr* as a new project
- Open within PyCharm: python/gps/algorithm/algorithm_traj_opt.py
- **Task: Implement the forward and backward pass of an LQR! Look at the website for advices`: https://goo.gl/X5twgi**

- You can test your implementation with a little test program
  - using a terminal, open the directory *ss2017_task1_lqr*
  - Start the program with: `python python/gps/lqr_test.py`
  - Was it successful?

# Thanks for your attention!

# Gaussians

- Univariate Gaussian
- Multivariate Gaussian
- Law of Total Probability
- Conditioning (Bayes' rule)

- Disclaimer: lots of lienar algebra in next few lectures. In fact, pretty much all computations with Gaussians will be reduced to linear algebra!